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Characterization of instrument spectral resolution by the spectral modulation transfer function

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ABSTRACT

Comparison of the spectral resolving power of different types of spectrometers can be facilitated by the use of the spectral modulation transfer function (SMTF). It is the Fourier transform of the instrument spectral response function. The SMTF s of the two imaging spectrometers included in the Advanced Land Imager of the EO-1 satellite are shown as examples. The SMTF offers a way to specify the spectral resolution required for the detection of known spectral features. It can also be used to determine the ultimate resolution that can be achieved through processing signals from any given instrument and observation.

Keywords: spectral resolution, spectrometer, hyperspectral imaging, MTF, performance

1. INTRODUCTION

Designers of instruments for remote sensing applications are often faced with a need to make compromises between performance specifications and cost factors. A thorough understanding of the performance of a proposed instrument and how it relates to the mission goals is essential if an optimum design is to be achieved. The subject of this paper is a key performance specification of optical spectrometers, namely spectral resolution. It is proposed here that it can be fully characterized by the spectral modulation transfer function (SMTF).

Imaging instruments have long been described and specified in terms of the modulation transfer function (MTF). The separate contributions of the optics, image detector (e. g. film or CCD), and following components such as amplifiers and image displays can be described and understood by their individual MTF s, which when multiplied together, yield the system MTF. For digital sensors, which inherently employ discrete sampling, the MTF of the system up to the point of sampling must be understood in relation to the Nyquist frequency (half the sampling frequency), in order to control aliasing errors.

1.1. Specification of Required Resolution

The spectral resolution required for a given type of observation or measurement depends on the source spectrum to be observed, the background spectra, and the required signal-to-noise ratio. If the objective is to measure the strength of a source with a known spectrum, there is little to be gained by having an instrumental resolution finer than the distinguishing features of the source spectrum. It is suggested that the Fourier transform of the known spectrum be examined to establish its true frequency content, in comparison with the background spectra. Signal-to-noise ratios should be estimated in relation to the instrument SMTF, in order to find the lowest resolution that will do the job. This should be done to minimize the cost associated with collecting aperture, data rate, and system complexity.

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1.2. Comparison of Instruments

The SMTF can also be a useful tool in comparing different types of spectrometers. Typically, the spectral sampling intervals are quoted in specifications. The shapes of the respective spectral response functions can be very different, however. This complicates any comparison of instruments. The SMTF clarifies the different resolutions, while allowing for all differences in spectral response functions.

In case measured spectra of the same scene are obtained with two different instruments, the SMTF facilitates cross-comparison as follows: Each spectrum is first Fourier transformed. The spectrum from the instrument with purportedly higher resolution is then degraded to the resolution of the other instrument by multiplying its transform by a filter formed as the ratio of the respective SMTF s. Upon inverse transformation, the spectra ought to be directly comparable.

2. SPECTRAL RESPONSE FUNCTIONS

The spectral response function is the apparent output of the instrument when stimulated by a monochromatic signal. It is analogous to the point-spread function of an imaging system. It can have various shapes for different types of spectrometers. It is generally a convolution of several functions representing the effects of diffraction, aberrations, slit or detector width, integration time, filter passband shape, and in the case of interferometers, maximum OPD. We shall next examine these components for the four types of spectrometers typically employed for remote sensing: grating and prism dispersive instruments, wedge filters, and interferometers.

2.1. Grating and prism spectrometers

A diffraction grating deflects photons from the zero-order path by an amount that is almost linearly dependent on wavelength. The optics of the spectrometer form an image of the entrance slit at the exit slit or detector plane. The response of a prism spectrometer is very similar to that of a grating spectrometer, except that the dispersion is a function of wavelength. Over a small range of wavelengths, the same type of analysis can be employed.

The optical line-spread function within the instrument represents the effects of diffraction and aberrations. Imperfections in a diffraction grating can cause artifacts such as side-lobes in the line-spread function. Slits or detector widths are represented by rectangle functions of some wavelength-equivalent width. For this discussion, the detector is assumed to have a broadband response, which does not influence the spectral response function. The integration time in spectrum-scanning instruments is analogous to a slit-width function.

2.2. Variable Filter spectrometers

Another type of spectrometer employs a circular or linear variable filter. An example is the Wedge Imaging Spectrometer (WIS) originally planned for the Advanced Land Imager (ALI) of the New Millennium Earth Orbiting-1 (EO-1) mission. That instrument had linear variable filters in close proximity to two-dimensional image detector arrays. Each detector row senses a particular wavelength. As the array executes a push-broom scan along the array columns, the spectrum of each ground pixel is sampled by the successive rows.

For variable-filter instruments, the passband shape of the filter depends on the details of its design. A narrow-band filter normally has a Lorentzian shape function, raised to the nth power, where n is the number of cavities. The width of the detector is a factor, as it is in a grating instrument. In case the variable filter spectrometer performs a scan over the spectrum, the integration time for each spectral sample also broadens the spectral response function.

2.3. Interferometric spectrometers

Interferometric spectrometers split the incoming beam in two, then recombine it after introducing an optical path difference between the beams. The detected interference pattern is recorded as a function of the optical path difference. The result is the Fourier transform of the spectrum, as expressed in wavenumber units (cm^{-1}).

3. SPECTRAL MTF S

The spectral modulation transfer function (SMTF) is defined as the Fourier transform of the spectral resolution function (normalized to unity at zero frequency). To evaluate the SMTF, we take advantage of the convolution theorem in Fourier transforms:[1]

$$f(x) * g(x) \leftrightarrow F(s)G(s) \quad , \tag{1}$$

where *F(s)* is the Fourier transform of *f(x)*, etc., and * represents a convolution. The scaling theorem is also indispensable:

$$f(ax) \leftrightarrow \frac{1}{|a|} F\left(\frac{s}{a}\right) \quad . \tag{2}$$

The SMTF is the product of the transforms of the various functions that are convolved together in the spectral resolution function.

3.1. Components of SMTF

The rectangle functions ($\Pi(x)$) representing slit width, detector width, or integration time all transform to sinc functions ($\sin(\pi s)/\pi s$):

$$\Pi(x) \leftrightarrow \text{sinc}(s) \quad . \tag{3}$$

The spectral variable, either wavelength or waveumber, is represented by x, and the spectral-frequency variable in the transform domain is represented by s. Optical effects within the spectrometer are represented by an optical MTF, expressed in wavelength-equivalent units. Aberration effects can be approximated by a Gaussian function multiplying the diffraction-limited MTF.[2] The Lorentzian shape of a single-cavity narrowband filter transforms to $\exp(-|s|)$:

$$\frac{2}{1+(2\pi x)^2} \leftrightarrow e^{-|s|} \quad . \tag{4}$$

If measured point transmission curves of a variable filter are available, it is best to perform a Fourier transform on them numerically.

1.2. Interferometer SMTF

The SMTF of an ideal interferometric spectrometer is simply a rectangular function, constant out to the maximum optical path difference employed. This is often modified by the application of a windowing function applied to the interferogram data.

Following the development of the Michelson interferometer, the narrowness of spectral line features was described by the *visibility curve*.[3] It gives the fringe contrast as a function of optical path difference. Since the interferogram is the Fourier transform of the spectrum, the visibility curve is mathematically the same as the SMTF, but the SMTF describes the instrument response, rather than a spectrum to be observed.

1.3. Critical Sampling

According to the Nyquist theorem, if a function (or signal) is sampled at a frequency that is greater than twice the highest frequency in its Fourier transform, it can be perfectly reconstructed from the samples. Conversely, sampling at a lower frequency results in *aliasing* errors.

The frequency content of the spectral signal from a given observation is the product of the transform of the source spectrum (the spectral spectrum) and the SMTF of the instr ument. If the instrument has a definite cutoff frequency (produced, for example, by diffraction), then there is a *critical sampling frequency*, which is twice the cutoff frequency. Sampling at a higher frequency produces superfluous data. Sampling at a lower frequency causes aliasing errors.

4. ADVANCED LAND IMAGER INSTRUMENTS

As examples we may use the imaging spectrometers originally planned for the Advanced Land Imager (ALI) of the New Millennium Earth Orbiting-1 (EO-1) mission.[4] They were the Wedge Imaging Spectrometer (WIS) and the Grating Imaging Spectrometer (GIS). Recently, those spectrometers were eliminated from the mission for programmatic reasons.

4.1. Grating SMTF

The sampling intervals of the GIS were 6 nm in the 400 to 1,000 nm (VNIR) range, and 12 nm in the 900 to 2,500 nm (SWIR) range. A beamsplitter was designed to direct these two spectral ranges to different focal plane arrays. (See Figure 1.) The SWIR array detected the first order, while the VNIR array used the second order. The widths of the pixels corresponded to the sampling intervals (with 100% fill factors). The entrance slit width (40 m) also matched those widths. The optics of the GIS matched the ˜-number of the telescope, 7.53.

The factors in the SMTF of the GIS are the optical MTF, the slit MTF (a sinc function), and the detector aperture MTF (the same as the slit MTF). These components and their product are plotted in Figure 3. The system SMTF falls to 50% at 70 cycles/ m, for λ ˉ 800 nm.

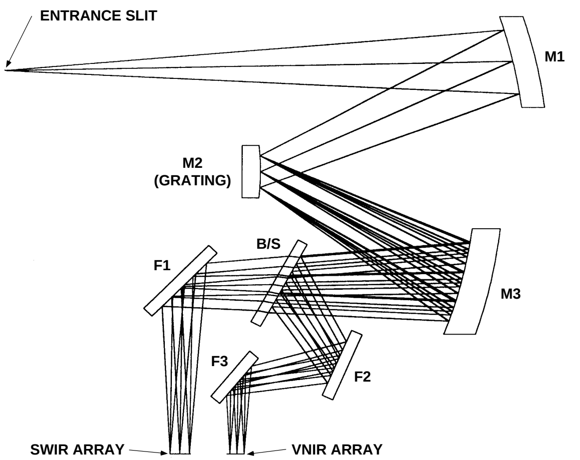

Figure 1. Optical layout of the Grating Imaging Spectrometer (GIS) planned for the EO-1 Advanced Land Imager.

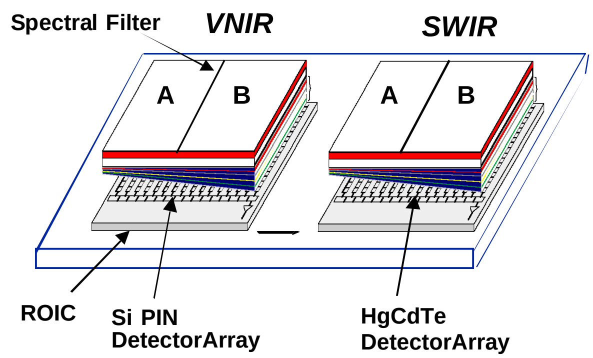

Figure 2. General arrangement of the Wedge Imaging Spectrometer (WIS) planned for the EO-1 Advanced Land Imager.

1.2. WIS SMTF

The WIS was designed for the same type of focal plane arrays as employed in the GIS. The linear variable (wedge) filters varied in the direction of the push-broom scan. (See Figure 2.) There were two filters each for the VNIR and SWIR arrays, since each filter could only be made to cover a free spectral range of approximately 1.65:1 in wavelength, while blocking radiation outside that range. Various spectral sampling intervals were used, from 6.5 nm in the VNIR range, to 10.3 nm in the 1,520 to 2,500 nm range.

The factors in the SMTF of the WIS are the point SMTF of the wedge filters, the aperture MTF caused by the extent of the ˜/7.53 light cone at the filter, and the detector aperture MTF.

An idealized wedge filter has a constant resolving power ($\lambda/\Delta\lambda$). Thus the SMTF varies with wavelength. We have received spectral transmission measurement data for some of the wedge filters produced by Raytheon Santa Barbara Remote Sensing for EO-1. They have been used to compute the filter SMTF shown in Figure 4. The other SMTF factors and the system SMTF are also plotted there. The system SMTF falls to 50% at 28 cycles/ m, for λ ˉ 800 nm.

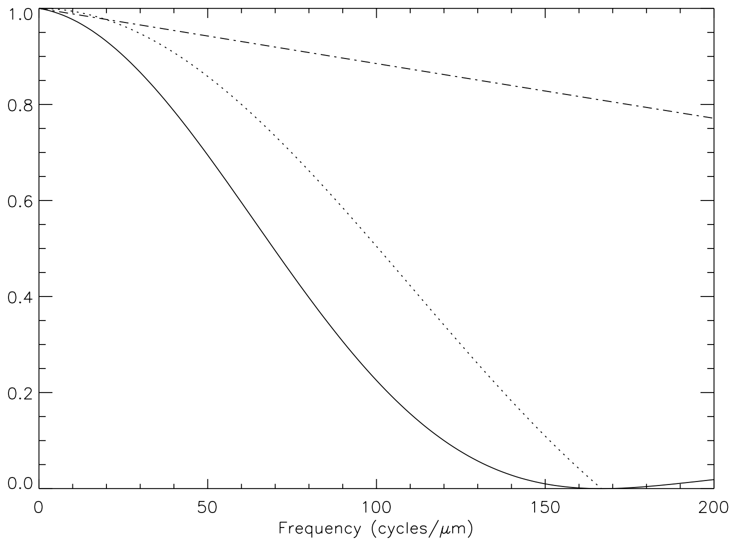

Figure 3. Spectral MTF (SMTF) of the Grating Imaging Spectrometer, for $\lambda = 800$ nm (solid curve). Contributing factors are also plotted: pixel and slit MTF s (dotted curve), and optical diffraction MTF (dot-dash curve). The Nyquist frequency is 83.33 cycles/ m .

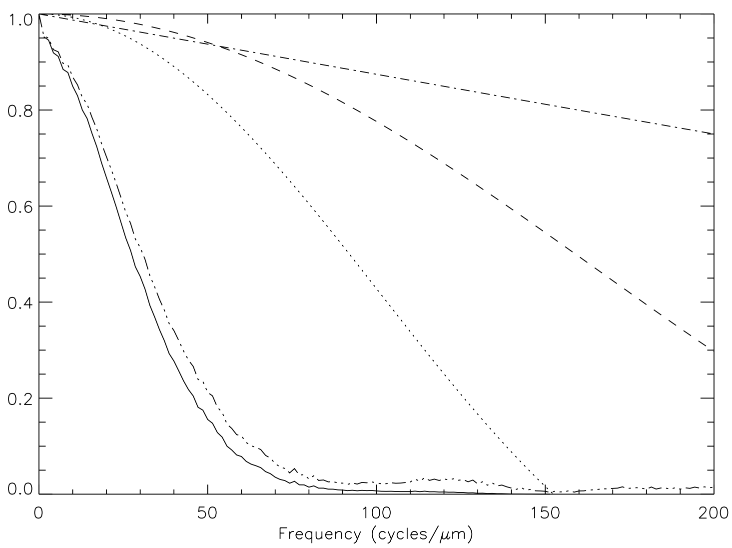

Figure 4. Spectral MTF (SMTF) of the Wedge Imaging Spectrometer, for $\lambda = 800$ nm (solid curve). Contributing factors are plotted: filter point SMTF (dash-dot-dot-dot curve), pixel MTF (dotted curve), optical diffraction MTF (dot-dash curve), and MTF of the focus cone spread at the filter (dashed curve). The Nyquist frequency is 76.22 cycles/ m .

Comparison of the two system SMTF curves in the figures clearly shows that, in this region of the spectrum, the GIS has approximately 2.5 times higher resolution than the WIS. (The two are more nearly equal in other spectral regions.)

5. SCENE SPECTRA

The utility of the SMTF is directly related to knowledge of the spectra to be observed. In Figure 5 are shown laboratory spectra of dolomite and grass, representing minerals and plants. Figure 6 shows the corresponding power spectral densities of the spectra. (The original spectral data have been re-sampled in 2 nm steps, with cubic spline interpolation, in order to perform FFT s on them.) We observe that for these examples, there is not much power in the transforms above a frequency of about 50 cycles/ m. Therefore, little is to be gained by building an instrument with a much broader SMTF. Of course, the spectra relevant to each particular mission must be examined in a similar way.

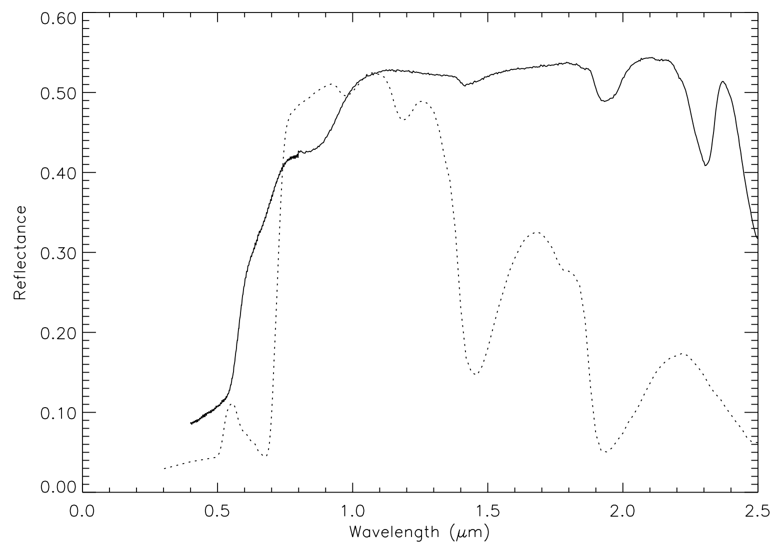

Figure 5. Laboratory reflectance spectra of Dolomite (solid curve) and green grass (dotted curve).

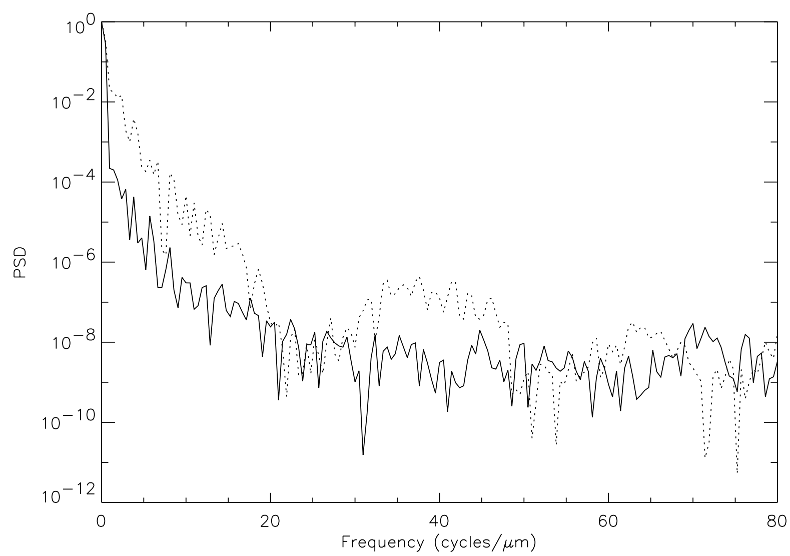

Figure 6. Power spectral densities of spectra shown in Figure 5: Dolomite (solid curve) and green grass (dotted curve). The PSD s are normalized to 1.0 at zero frequency (representing unity total power in the spectrum).

6. MEASURES OF RESOLUTION

Just as with spatial resolution, there are many ways of describing spectral resolution. Some are convenient and easy to understand, but may not convey a full understanding of the true capabilities of an instrument. It is particularly risky to compare two different kinds of instrument by some simple criterion, such as spectral sampling interval. In the spectral domain, it is better to use the full width at half-maximum (FWHM) of the spectral response function. The SMTF gives a complete description of those capabilities. It is desirable though, to find a simple number, or figure of merit, for comparing instruments.

Several meaningful figures of merit may be found from the SMTF. The value of the SMTF at the Nyquist frequency (half the sampling frequency) is one such number. Some other reference frequency could also be used. Instead of specifying SMTF (contrast) at a given frequency, we could invert the process and specify the frequency at which a given contrast is achieved. It could be 50%, 5%, or the noise-to-signal ratio. The reciprocal of that frequency represents the corresponding spectral resolution. This appears to be a very general approach to comparing spectrometric instruments.

6.1. Resolution Enhancement

Under favorable circumstances, it is possible to enhance the resolution of an instrument through appropriate data processing. If the signal-to-noise ratio is high enough, and the response function has been characterized well enough, a Wiener filter can be constructed, to restore frequency components attenuated by the SMTF of the instrument within limits imposed by the noise.[5] The form of a Wiener filter is

$$W = \frac{S}{(S+N)M_S} \quad , \tag{5}$$

where

S = the detected signal power spectral density (PSD),

N = the measurement noise PSD, and

M_S = the (spectral) modulation transfer function (SMTF).

The Wiener filter is greater than one for frequencies that are attenuated by the instrument, as long as the signal to noise ratio is large. Otherwise, the filter becomes less than one as noise dominates the signal.

The signal PSD, S, is the product of the PSD of the source spectrum (P_S) and the squared SMTF of the instrument:

$$S = {M_S}^2 P_S \quad .$$

Using this relationship, the Wiener filter can be expressed as

$$W = \frac{M_S P_S}{{M_S}^2 P_S + N} = \left(M_S + \frac{N}{M_S P_S} \right)^{-1} \quad . \tag{6}$$

The ultimate spectral resolution possible for a given instrument and observation can be estimated from the SMTF and the signal-to-noise ratio (SNR). We shall take the ultimate frequency in the transform domain, s_U, to be that at which W equals one half:

$$W(s_U) = 0.5 \Rightarrow M_S(s_U)\left[2 - M_S(s_U)\right] = \frac{N}{P_S(s_U)} \quad . \tag{7}$$

The SMTF of an ideal instrument is one, and it can not be greater than one. If P_S is much greater than N for all frequencies, then the ultimate frequency is the solution of $M_S(s_U) = 0$. If, on the other hand, the source spectrum (P_S) is limited in frequency extent, then equation 7 can be used in optimizing the instrument design in a cost *vs*. performance trade. In any case, it is essential to have a reasonable estimate of the signal-to-noise ratio in order to find the true limits to resolution.

7. SUMMARY

The spectral modulation transfer function (SMTF) is a powerful tool for analyzing the performance of a spectrometer in relation to any given type of observation. It can be particularly useful in arriving at a design that returns the most performance in a given mission for the least cost. It can also provide an unbiased basis for comparing the performance of different kinds of spectrometers.

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